Atmospheric Dynamics I

1d. Determine kinematic processes which describe vorticity in wind flow.

Vorticity

- Measure of the local rotation (i.e., spin) of atmospheric parcels
- Measured in units if radians
- Radians per revolution: 1 rev = 2π radians = 2*(3.14…) radians = (6.28…) radians

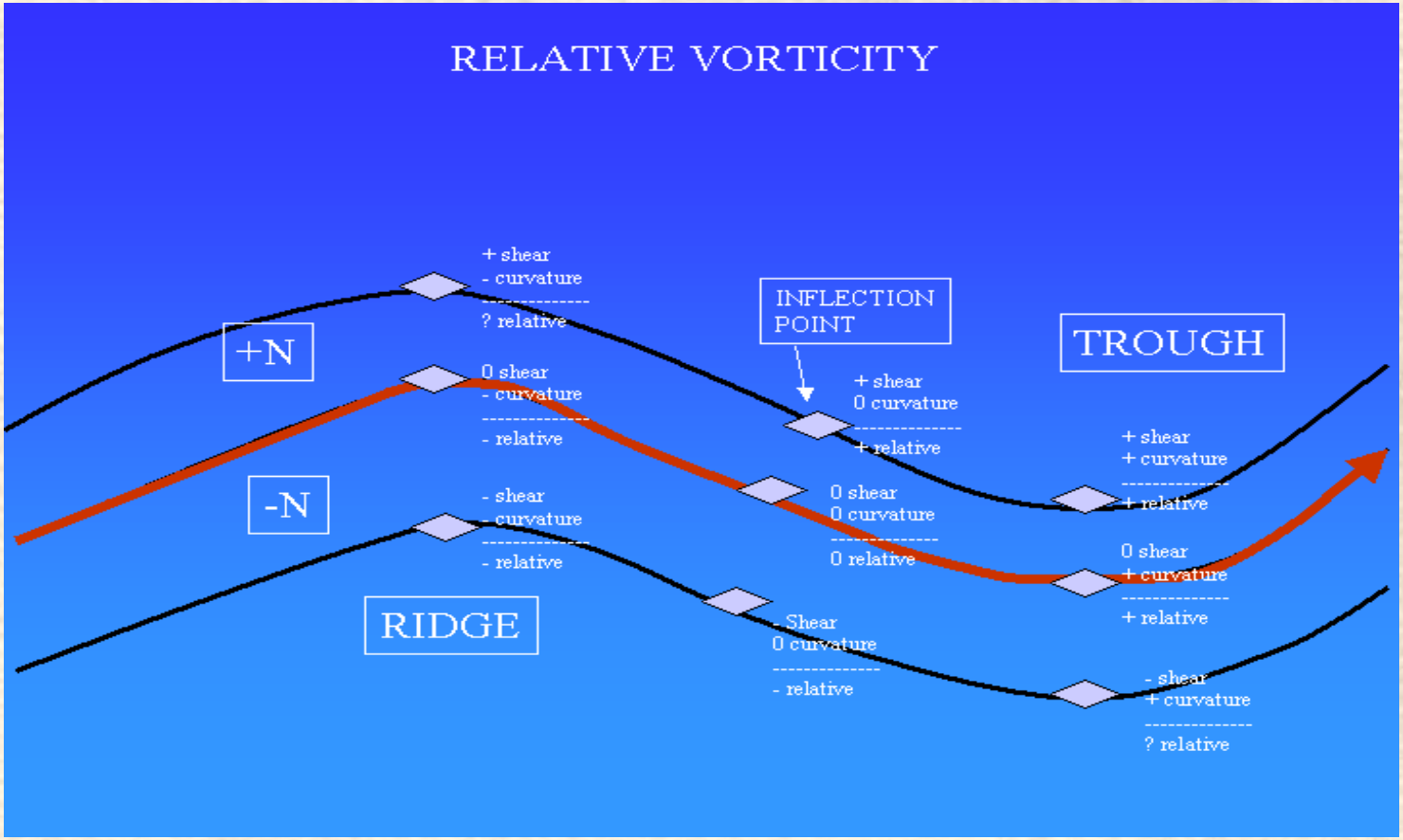
Vorticity

- Positive vorticity: cyclonic spin
- Negative vorticity: anticyclonic spin
- Positive vorticity advection: Higher values of vorticity replacing lower values of vorticity through horizontal advection
- Negative vorticity advection: Lower values of vorticity replacing higher values of vorticity through horizontal advection

Types of Vorticity

- Relative Vorticity
  - Definition: Sum of both curvature vorticity and shear vorticity.
    - Measure local spin of the air RELATIVE to the earth. (Moving with the coordinate system fixed to the earth)

Relative vorticity = Curvature and Shear



CURVATURE VORTICITY

Vorticity Film

Types of Vorticity

- Equation: Relative vorticity (ζ) = curvature + shear. Stated mathematically:

Relative vorticity Equation in Natural Coordinates (Vertical only)

$$\zeta = V \frac{\Delta\alpha}{\Delta s} - \frac{\Delta V}{\Delta n}$$

(A) **(B)**

Types of Vorticity

$$\zeta = V\,\frac{\Delta\alpha}{\Delta s} - \frac{\Delta V}{\Delta n}$$

(A) **(B)**

- Term A represents the change in wind direction Δα along the wind flow (Δs). Thus, Term A represents the curvature of the wind flow in a cyclonic, anticyclonic, or straight (i.e., curvature) nature. In Term A:
  - V is the wind speed
  - Δα is the angle of change in the wind direction along the flow in the + Δs direction (i.e., Δs is positive in the direction to the left of the wind flow).

Types of Vorticity

$$\zeta = V\frac{\Delta\alpha}{\Delta s} - \frac{\Delta V}{\Delta n}$$

(A) **(B)**

» Winds backing along +Δs (i.e.,turning in a counter-clockwise “cyclonic” direction along +Δs) produce a “positive” change in Δα (i.e., Δα is “positive”

» Winds veering along +Δs (I.e. turning in a clockwise “anticyclonic” direction along +Δs) produce a “negative” change in Δα (i.e., Δα is “negative”)

Types of Vorticity

- Relative Vorticity cont. $\zeta = V \frac{\Delta\alpha}{\Delta s} - \frac{\Delta V}{\Delta n}$

(A) **(B)**

  - Term B represents the change in wind speed (ΔV) across the path of the wind flow (Δn represents a distance increment across the flow). Thus, Term B represents the horizontal shear of the wind flow (or speed divergence).

Types of Vorticity

- Relative Vorticity $\boxed{\frac{\zeta}{\Delta s}} = V\frac{\Delta\alpha}{\Delta n} - \frac{\Delta V}{\_\_}$

  (A) **(B)**

  - Term A (Curvature Vorticity)
    - Definition: Vorticity resulting from the curvature of the wind flow.
      - Positive (+) in regions of cyclonic turning.
      - Negative (-) in regions of anticyclonic turning
      - Zero at inflection points (NO curvature)

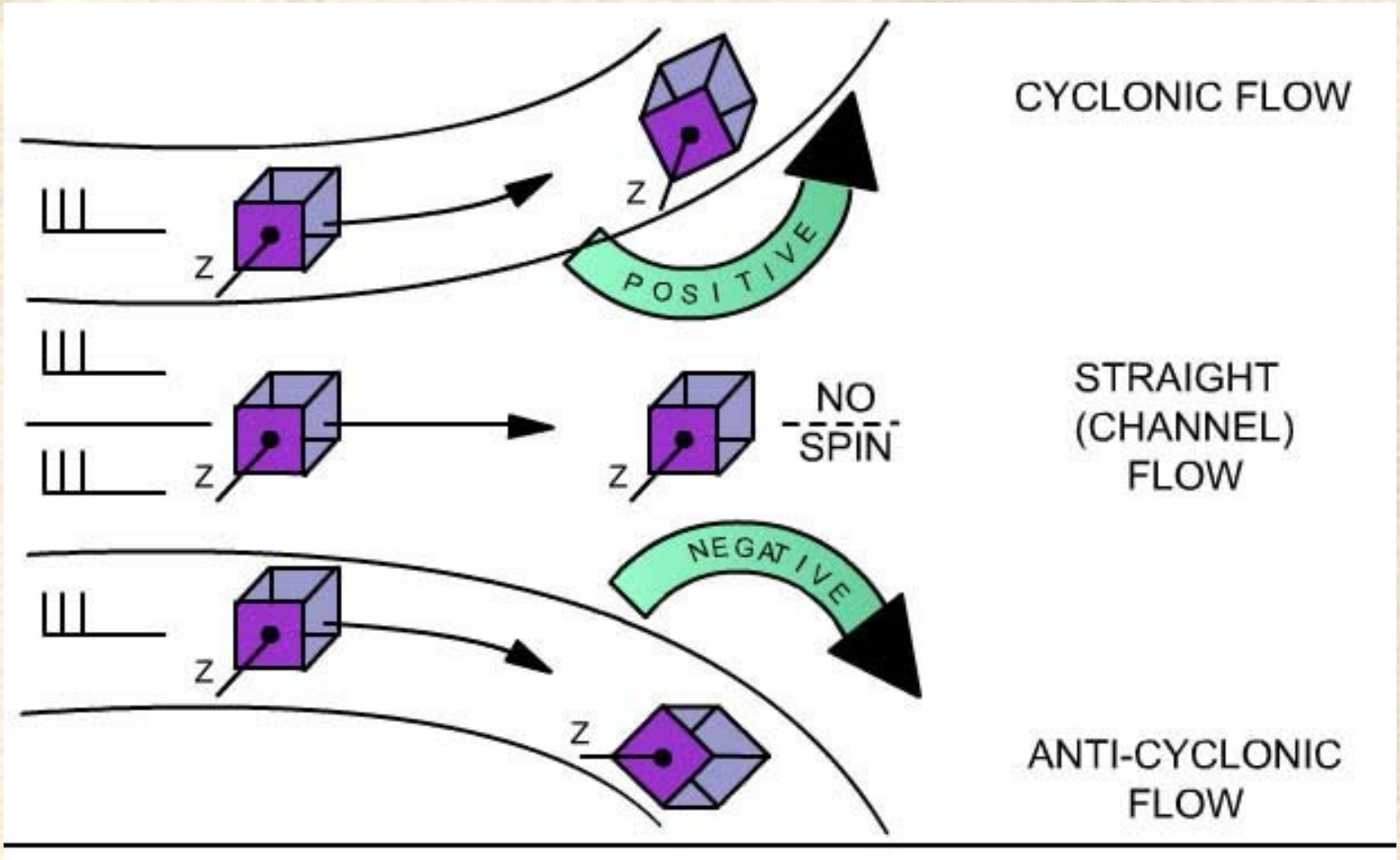
Types of Vorticity

- Curvature Vorticity
  - Cyclonic Curvature Vorticity

$$\mathbf{V}\,\frac{\Delta\alpha}{\Delta s} = V\,\frac{(\alpha_{downstream} - \alpha_{upstream})}{(s_{downstream} - s_{upstream})} = +\,\frac{(larger - smaller)}{(larger - smaller)} = +\,\frac{(+)}{(+)} = +$$

    - The more the wind direction “backs” along a given path distance increment “+Δs”, the greater the cyclonic curvature of the flow.
    - (PPT **example** (Not available) depicts weaker vs. stronger backing of the wind and the resulting weaker vs. stronger cyclonic curvature vorticity)

Curvature vorticity



Types of Vorticity

- Curvature Vorticity cont.
  - Anticyclonic Curvature vorticity

$V\,\underline{\Delta\alpha} = V(\underline{\alpha_{downstream} - \alpha_{upstream}}) = +\,(\underline{smaller - larger}) = +(\underline{-})$

$= -$

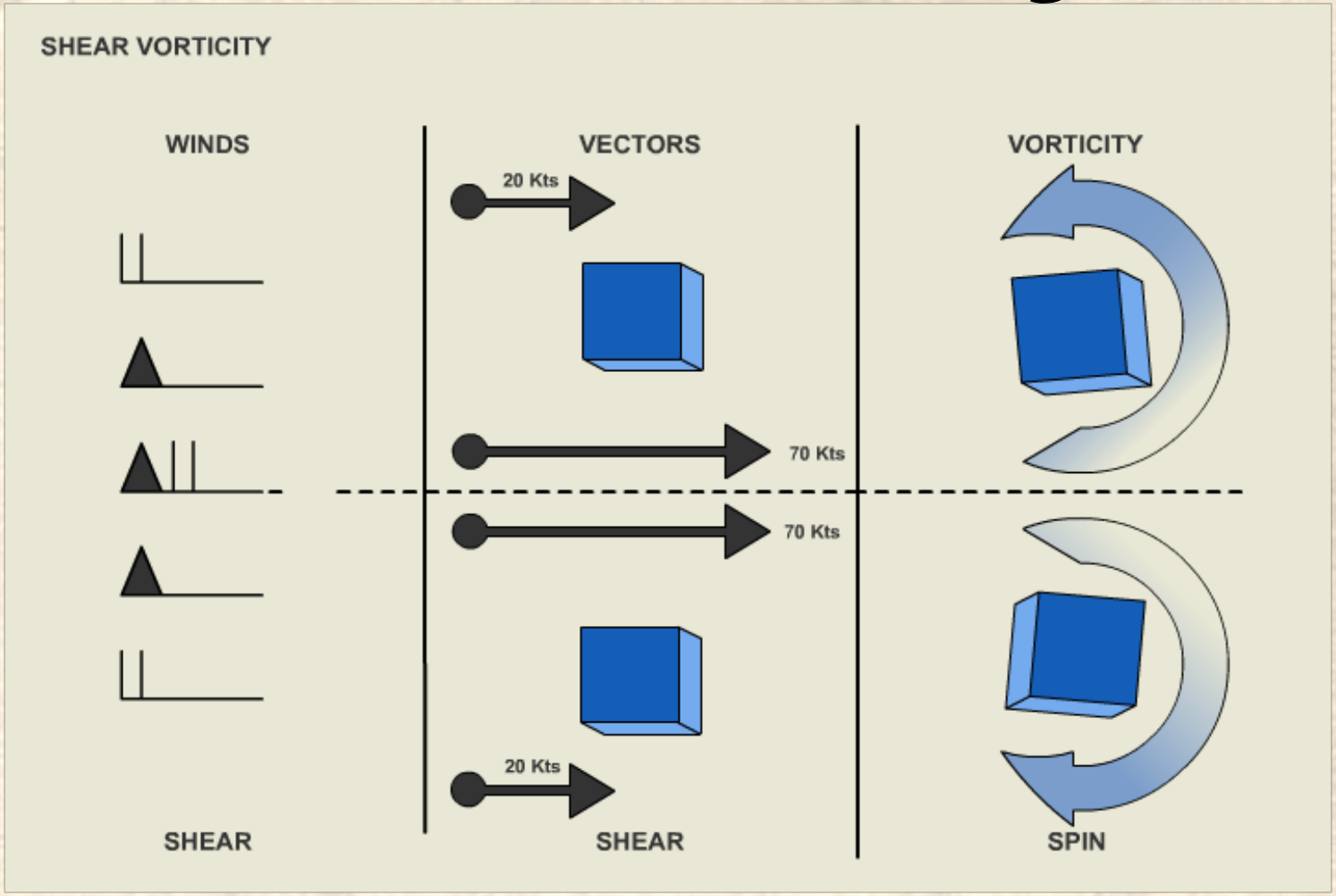
$\Delta s \qquad (s_{downstream} - s_{upstream}) \qquad (larger - smaller) \qquad (+)$

- The more the wind direction “veers” along a given path distance increment “+Δs”, the greater the anticyclonic curvature of the flow.
- (PPT example depicts weaker vs. stronger veering of the wind and the resulting weaker vs. stronger anticyclonic curvature vorticity.)

Types of Vorticity

- Relative Vorticity $\zeta = V \left[\frac{\Delta\alpha}{\Delta s} - \frac{\Delta V}{\Delta n} \right]$ **(A)** **(B)**
  - Term B (Shear Vorticity)
    - Definition: Vorticity resulting form the lateral speed shear in the wind flow.
      - Positive (+) in the +n direction from the fastest wind.
      - Negative (-) in the –n direction from the fastest wind.

Shear vorticity



Types of Vorticity

- Shear vorticity
  - Cyclonic Shear Vorticity

$$-\Delta V = -(V_{\text{left of flow}} - V_{\text{right of flow}}) = -(\text{smaller} - \text{larger}) = -(-) = +$$

$$\Delta n \quad (n_{\text{left of flow}} - n_{\text{right of flow}}) \qquad (\text{larger} - \text{smaller}) \quad (+)$$

- The greater the wind speed decrease across the flow for a given distance increment "+Δn", the greater the cyclonic shearing of the flow.

Types of Vorticity

- Shear Vorticity
  - Anticyclonic Shear Vorticity

$$-\frac{\Delta V}{\Delta n} = -\frac{(V_{\text{left of flow}} - V_{\text{right of flow}})}{(n_{\text{left of flow}} - n_{\text{right of flow}})} = -\frac{\text{(larger - smaller)}}{\text{(+)}} = -\frac{(+)}{(+)} = -$$

    - The greater the wind speed increase across the flow for a given distance increment "+Δn", the greater the anticyclonic shearing of the flow.
    - (PPT example (Not available) depicts weaker vs. stronger anticyclonic shearing and resulting weaker vs. stronger anticyclonic shear vorticity)

Types of Vorticity

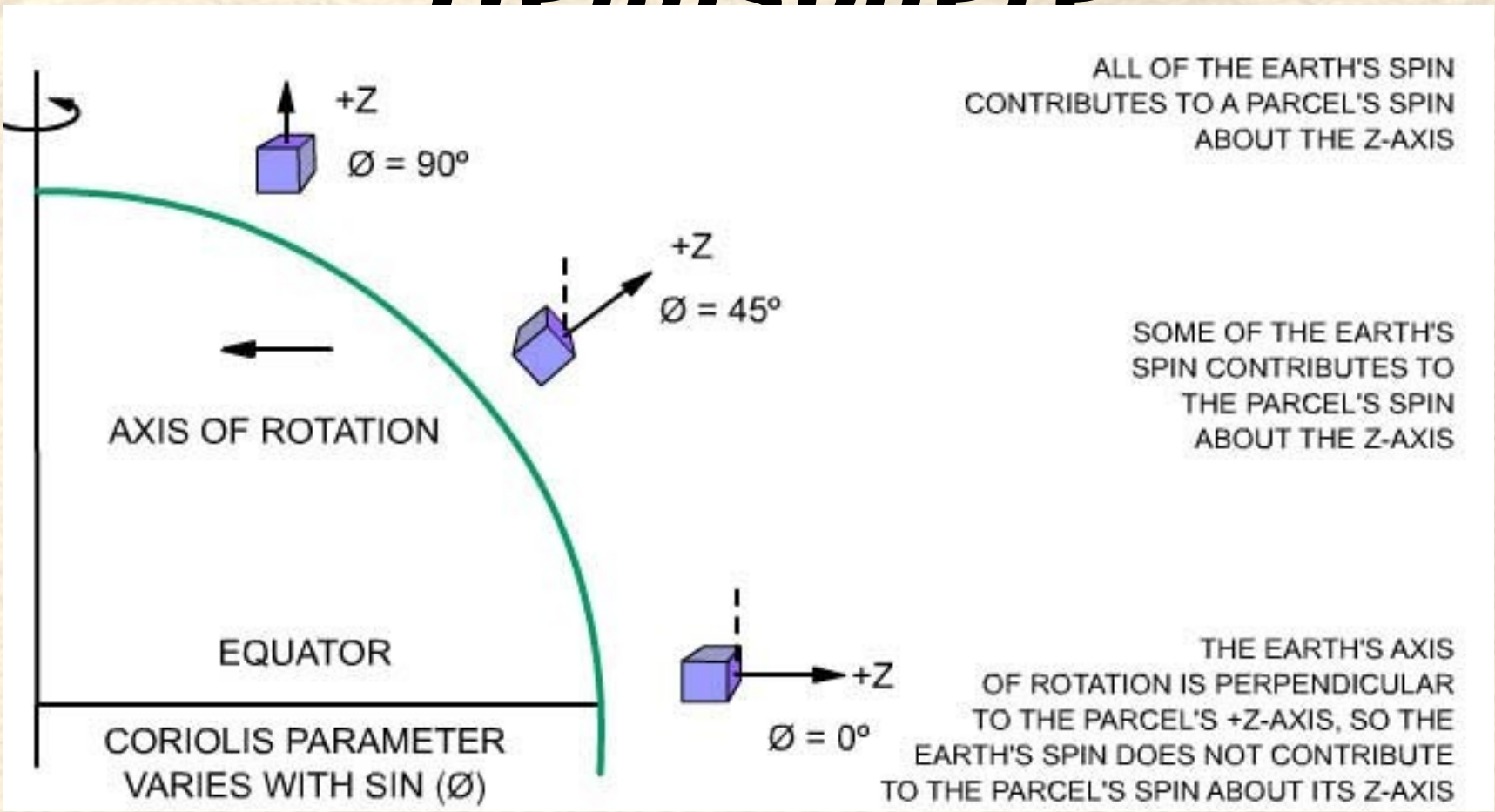
- Planetary Vorticity
  - Definition: The local spin provided to the atmosphere by the earth's rotation
    - The Coriolis parameter "f" is a measure of the amount of planetary vorticity. ($f = 2\Omega\sin\theta$) (See previous discussions in Sections a and b of Dynamics I)

Types of Vorticity

- Planetary Vorticity
  - Examples: $f = 2\Omega\sin\theta$, for a given latitude;

Latitude	θ	Coriolis Parameter (f)
90°N	+90°	14 X 10^{-5} radians/sec
40°N	+40°	9 X 10^{-5} radians/sec
30°N	+30°	7 X 10^{-5} radians/sec
0^{o}	0°	0
30°S	-30°	- 7 X 10^{-5} radians/sec
40°S	-40°	- 9 X 10^{-5} radians/sec
90°S	-90°	- 14 X 10^{-5} radians/sec

Latitudinal variation of f in the Northern Hemisphere


+Z
Ø = 90°
ALL OF THE EARTH'S SPIN CONTRIBUTES TO A PARCEL'S SPIN ABOUT THE Z-AXIS
+Z
Ø = 45°
SOME OF THE EARTH'S SPIN CONTRIBUTES TO THE PARCEL'S SPIN ABOUT THE Z-AXIS
AXIS OF ROTATION
EQUATOR
+Z
Ø = 0°
THE EARTH'S AXIS OF ROTATION IS PERPENDICULAR TO THE PARCEL'S +Z-AXIS, SO THE EARTH'S SPIN DOES NOT CONTRIBUTE TO THE PARCEL'S SPIN ABOUT ITS Z-AXIS
CORIOLIS PARAMETER VARIES WITH SIN (Ø)

Types of Vorticity

- Absolute Vorticity
  - Definition: **The total spin of an air parcel** as observed in from an ABSOLUTE (star-based) reference frame.

Absolute Vorticity (η) = Relative Vorticity (ζ) + Planetary Vorticity (f)

= (Curvature Vorticity + Shear Vorticity) + Planetary Vorticity

Types of Vorticity

- Absolute Vorticity cont.
  - Stated mathematically:

    Absolute Vorticity Equation in Natural Coordinates

    (Vertical Only)

$$\eta = (\zeta + f) = V\frac{\Delta\alpha}{\Delta s} - \frac{\Delta V}{\Delta n} + f$$

(A) (B) (C) (D) (E)

Types of Vorticity

$$\eta = (\zeta + f) = V\frac{\Delta\alpha}{\Delta s} - \frac{\Delta V}{\Delta n} + f$$

(A) (B) (C) (D) (E)

- Term A = relative vorticity
- Term B and E = planetary vorticity (Coriolis parameter)
- Term C and D = curvature vorticity and shear vorticity, respectively

Types of Vorticity

- Remember: The Coriolis parameter is **ALWAYS POSITIVE** in the Northern Hemisphere. However, its strength varies as the sine of the latitude.
- Coriolis parameter is typically large enough to result in a positive absolute vorticity value for all synoptic scale motions in the Northern Hemisphere.

Types of Vorticity

- By subtracting the Coriolis parameter from the absolute vorticity we obtain relative vorticity, which indicates the actual strength of the system. Mathematically: $\zeta = \eta - f$

Types of Vorticity

- When comparing troughs or lows with the same absolute vorticity value, the cyclonic relative vorticity, and therefore the trough or low, is stronger at lower latitudes.

Lat.	Ab Vort (η)	Rel Vort (ζ)	f
40°N	+20	11	9
30°N	+20	13	7

Types of Vorticity

- For a constant absolute vorticity value, as f decreases moving equatorward, cyclonic relative vorticity increases (large positive values)
- For a constant absolute vorticity value, as f increases moving poleward, cyclonic relative vorticity decreases (small positive values)

Types of Vorticity

– When comparing ridges or highs with the same absolute vorticity value, the anticyclonic relative vorticity, and therefore the ridge or high is stronger at higher latitudes.

Lat.	Ab Vort (η)	Rel Vort (ζ)	f
40°N	+4	-5	9
30°N	+4	-3	7

Types of Vorticity

- For a constant absolute vorticity value as f increases moving poleward, anticyclonic relative vorticity increases (larger negative values)

- For a constant absolute vorticity value f decrease moving equatorward, anticyclonic relative vorticity decreases (smaller negative values)

Types of Vorticity

– This conservation of absolute vorticity, and the variation of relative vorticity as planetary vorticity fluctuates with latitude, is an extremely important concept in explaining synoptic and planetary waves in the mid-latitude westerlies.

Types of Vorticity

- Potential vorticity is a measure of the potential spin rotation in an atmospheric layer.
- In a simplified mathematical form, potential vorticity (P) is the ratio of the absolute vorticity (η) to the effective depth of the layer (Δz).

Types of Vorticity

- In a simplified mathematical form, potential vorticity (P) is the ratio of the absolute vorticity (η) to the effective depth of the layer (Δz).

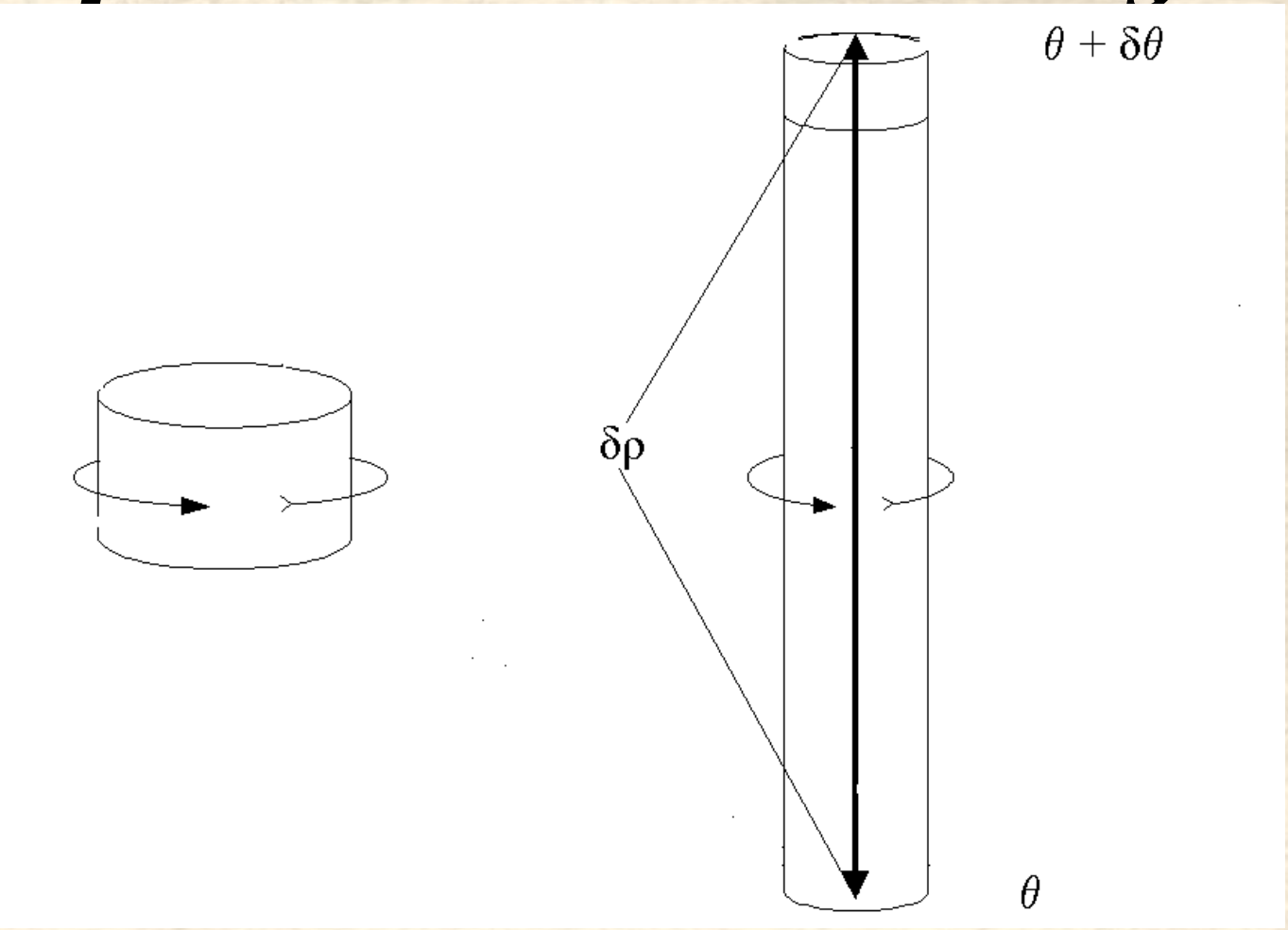
$$P = \frac{(\eta)}{(\Delta z)} = \frac{(\zeta + f)}{\Delta z} \quad \text{so} \quad \eta = P \Delta z$$

Types of Vorticity

$$P = \frac{(\eta)}{\Delta z} = \frac{(\zeta + f)}{\Delta z}, \text{ so } \eta = P(\Delta z)$$

- If P is kept constant, variations in depth of the layer can result in changes in the absolute vorticity.
  - If the layer is stretched vertically (i.e., Δz increases), then the absolute vorticity must increase, effectively strengthening the cyclonic circulation.

Conservation of potential vorticity



Types of Vorticity

- If the layer is compressed vertically (i.e., Δz decreases), then the absolute vorticity must decrease, effectively strengthening the anticyclonic circulation.
  - This also is an important process in lee-side troughing east of the Rocky Mountains as air "compresses" upward while ascending the west side of the mountain range.

Types of Vorticity

- Geostrophic Forms of Relative and Absolute Vorticity
  - Definition: The relative or absolute vorticity generated by the curvature and shear of the geostrophic wind.
  - Utility: Since the geostrophic wind blows:
    - Parallel to the height contours on an isobaric surface,and
    - With a speed proportional to the HGF…….⇒

Types of Vorticity

- …It can be shown mathematically that the height contour pattern on a constant pressure surface (such as the contours on the 500 mb pressure surface) can be used directly to calculate geostrophic vorticity, where:

Types of Vorticity

- Geostrophic relative vorticity (ζ_g) = curvature and gradient pattern of the height contours (Z) on an isobaric surface (e.g., 500 mb).

Geostrophic Relative Vorticity Equation

$$\zeta_g = V_g \frac{\Delta\alpha}{\Delta s} - \frac{\Delta V_g}{\Delta n} = \frac{g}{f} \frac{[Z_N + Z_E + Z_S + Z_W - (4*Z_C)]}{d^2}$$

(A) (B) (C)

Types of Vorticity

$$\zeta_g = V_g \frac{\Delta\alpha}{\Delta s} - \frac{\Delta V_g}{\Delta n} = \frac{g}{f} \left[\frac{Z_N + Z_E + Z_S + Z_W - (4 * Z_C)}{d^2} \right]$$

(A) (B) (C)

where:

- Term A = curvature vorticity of the geostrophic wind
- Term B = shear vorticity of the geostrophic wind
- Term C = the local distribution (i.e., Laplacian) of heights on an isobaric surface

where:

Types of Vorticity

$$\zeta_g = V_g \frac{\Delta\alpha}{\Delta s} - \frac{\Delta V_g}{\Delta n} = \frac{g}{f}\frac{[Z_N + Z_E + Z_S + Z_W - (4*Z_C)]}{d^2}$$

(A) (B) (C) where:

- Z_N= height at the northern point
- Z_E = height at the eastern point
- Z_S = height at the southern point
- Z_W = height at the western point
- d = an equal distance between each of the points (Z_N, Z_E, Z_S, and Z_W)
- g = gravity
- f = Coriolis parameter

Types of Vorticity

- Geostrophic Forms of Relative and Absolute Vorticity cont.
  - Geostrophic Absolute Vorticity (η_g) = Geostrophic Relative Vorticity (ζ_g) + Planetary Vorticity (f)

  $\forall \Rightarrow$ Stated mathematically:

  Geostrophic Absolute Vorticity Equation

$$\eta_g = (\zeta_g + f)$$

(A) (B) where:

Types of Vorticity

- Geostrophic Forms of Relative and Absolute Vorticity cont.

$$\eta_g = (\zeta_g + f)$$

(A) (B) where:

- Term A = geostrophic relative vorticity
- Term B = planetary vorticity (Coriolis parameter)

Types of Vorticity

- The geostrophic forms of vorticity are only estimates of the actual vorticity, and are based on the following assumptions:
  - The wind is in a geostrophic balance with the height field on an isobaric surface.
  - The geostrophic wind is a good approximation of the wind on the isobaric surface.

Types of Vorticity

– The geostrophic estimate of vorticity is an extremely powerful tool and is basis for the development of quasi-geostrophic theory of synoptic dynamics (discussed later in Dynamics I)..

Types of Vorticity

- Because of the relationship of geostrophic wind to the height pattern, a realistic estimate of vorticity can now be obtained strictly from the height contour pattern on an isobaric surface. Thus, a direct relationship between height contour and vorticity exists WITHOUT the necessity for explicit portrayal of winds.

  Quasi-geostrophic theory utilizes this concept toward development of powerful tools which help to link atmospheric dynamics to meteorological applications.

Types of Vorticty

– Thus, much dynamical insight is possible strictly by analyzing the height on an isobaric surface.

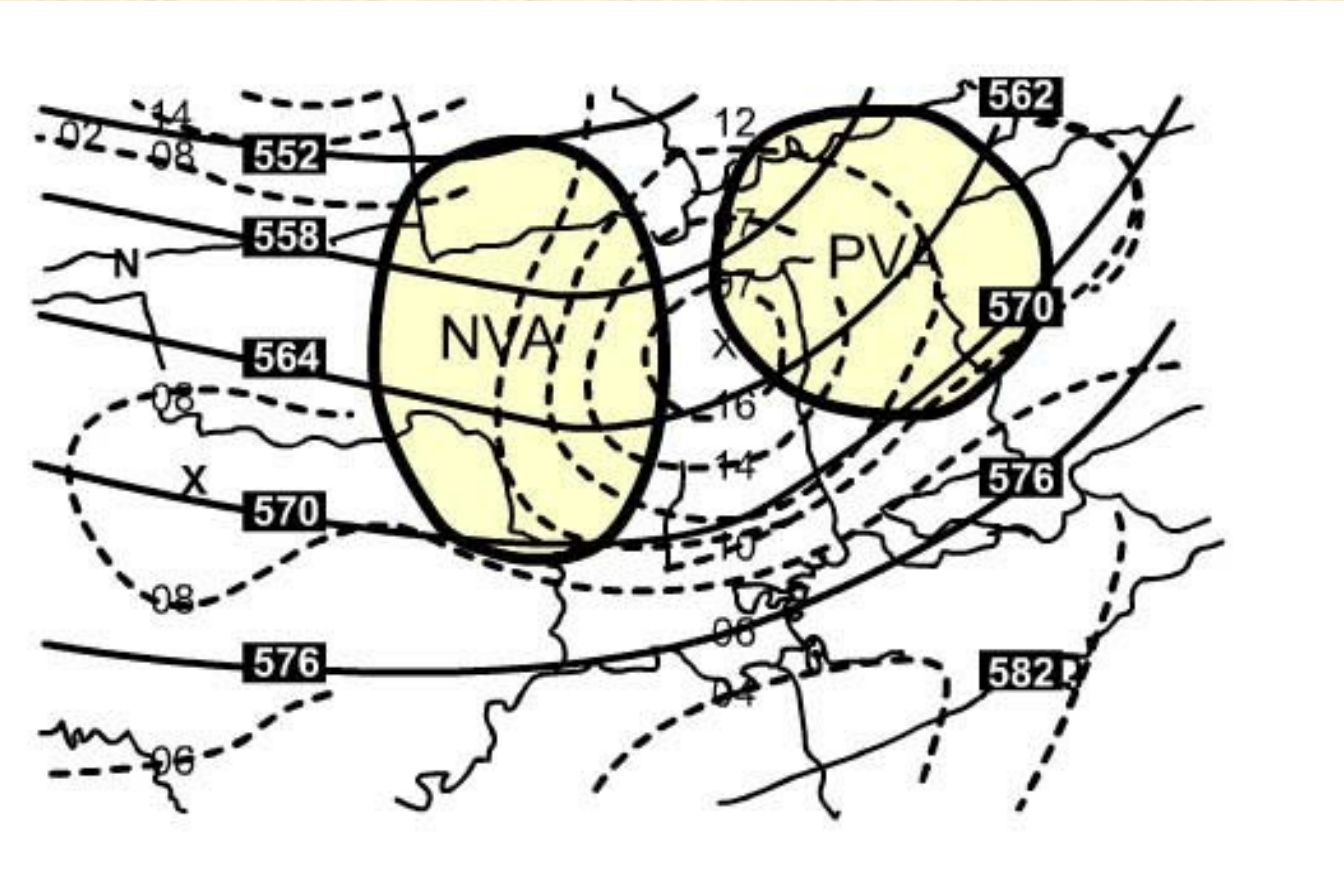
Advection of Absolute Vorticity

- Advection analysis is done using the 500mb heights/vorticity analysis
- Types of vorticity advection
  - Positive Vorticity Advection (PVA) at 500mb
    - ⇒ Increasing absolute vorticity with time at a location due to horizontal advection of higher vorticity into a region.

Advection of Absolute Vorticity

- Types of Vorticity Advection cont.
  - Negative Vorticity Advection (NVA) at 500 mb
    - Decreasing absolute vorticity with time at a location due to horizontal advection of lower vorticity into the region.
  - Neutral Vorticity Advection at 500 mb
    - No change of absolute vorticity with time at a given location due to no horizontal advection of vorticity into the region.

500 mb Vorticity chart and areas of PVA, NVA



Advection of Absolute Vorticity

- As will be shown in a later section in Dynamics I, quasi-geostrophic theory links (PVA/NVA) with (divergence/convergence) in the upper troposphere. This association has a great value in linking vertical motion to the upper-level wave and vorticity patterns. However, as also will be shown, many important assumptions govern the correct usage of this relationship.

- APPLICATION: Review Exercise 4
- EVALUATION: Informal: Appraisal
  Formal: Written Test